2017 Fall EE203001 Linear Algebra - Midterm 1 solution

1. (10%)

(a)

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(b) a=8

(c) a =8

(d) a=8 , b=100

(e) a=8 , b =100

2. (15%)

(a)

1 2 3 31 2 3 5 69 3 5 a b

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→

1 1 1 1 1 0 0 0 0 1 2 3 0 1 0 0 0 0 -5 -6 0 0 1 0 0 0 1 1 0 0 0 1

1 1 1 1 1 0 0 0 [

A I

]

=

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0 1 2 3 0 1 0 0 0 1 1 0 0 0 0 1 0 0 0 1 0 0 -1 -5

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A−1 =

1 0 0 0 1 -1 -1 -4 0 1 0 0 0 1 1 3 0 0 1 0 0 0 1 6

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0 0 0 1 0 0 -1 -5

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(b) A =

1 -1 -1 -4 0 1 1 3 0 0 1 6 0 0 -1 -5

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1 1  0 1 0 0 1 2 -5 1 2 -6

   0 0 1 1

[

B

2×2

]

[

B C I O O D O I

=

O

2×2

C

2×2 D

2×2

]

→

[

I B−1C B−1 O O I O D−1

]

→

[

I O B−1 -B−1CD−1 O I O D−1

]

A−1 =

[

B−1 -B−1CD−1

O D−1

]

(c) B−1 =

[

1 -1 0 1

]

,D−1 =

[

1 6 -1 - 5

]

- B−1CD−1 = -

[

1 -1 0 1

][

1 1 2 3

][

1 6 -1 - 5

]

=

[

-1 -4 1 3

]

A−1 =

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1 -1  0 1 0 0 [

B−1 -B−1CD−1

-1 1 1 ] O D−1

=

-4 3 6

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1

3. (20%)

(a) P

13

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E

21

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E

32

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PA = E−1

21

0 0 1

1 0 0

1 0 0 =

0 1 0

=

-2 1 0

=

0 1 0 1 0 0

0 0 1

0 -2 1

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= LU

(b) PAx =

1 0 0

1 1 2 E−1

32

U =

2 1 0 0 2 1

0 1 3

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0 

0 -2 

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⇒ LUx =

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c =

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⇒ c =

0

0

1 0 0 -2

-2

2 1 0

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0

-2 



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-2

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0 2 1

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-2

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x =

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⇒ x =

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4. (20%)

(a) Reduce A to either

1 1 2 0 1 3 0 0 -2

0 -2 2

1 1 -1

0 -2 -2

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or its RREF

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⇒ a basis for

Row(A) = 1(1,2,0,3),(0,1,3,0),(0,0,1,0)l or

1(1,0,0,3),(0,1,0,0),(0,0,1,0)l or 1(1,2,0,3),(2,5,3,6),(1,3,4,3)l

⇒ a basis for

Col(A) = 1(1,2,1)T,(2,5,3)T,(0,3,4)Tl .

(b) Yes, since rank(A)=3=m.

(c) No,since rank(A)=3 = n = 4

(d) rank(B) = n - dim(N(B)) = 4 - 3=1 dim(N(BT)) = 5 - rank(B)=4

5. (20%)

(a) [A|b] =

1 2 0 3 0 1 3 0 0 0 1 0

1 0 0 3 0 1 0 0 0 0 1 0

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1 2 -2 5 6 13 -11 31 2 4 -4 10 10 22 -18 52 -15 17

41 8

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     7 33 10 56 43 pivot variables : x

1

1 0 0 0 0 0 1 0 2 1 →

0 0 0 0 0 0 0 0 0 0 0 0 1 -1 1 , x

2

, x

3 free variables : x

4

(b) Find x

p

: set x

4

= 0, apply backward substitution to R x = b  



x

1

= 0 x

2

= 1 x

3

= 1

→ x

p

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0 1 1

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=

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2

(c) Find special solution : set x

4

= c, apply backward substitution to R x = 0  



x

1

= 0 x

2

+ 2x

4

= 0 x

3

- x

4

= 0

→ x

h

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(d) x

complete

0

= c

-2 1 1

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0 1 1

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   0

(e) dim(row(A)) = dim(col(A)) = 3

dim(N(A)) = 4 - rank(A)=4 - 3=1

dim(N(AT)) = 5 - rank(AT)=5 - rank(A)=5 - 3=2

(f) basis of col(A) : any three independent vectors in span1first three columns of Al

basis of row(A) : any three independent vectors in span1row 1,2,5 of Al

basis of N(A) : any vector in span1

0

= x

p

+ x

h

=

+ c

-2 1 1

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0

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-2 1

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basis of N(AT) : For N(AT), we can find E and R such EA = R

[I|A] =

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1 0  0 1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 | | | | 1 2 -2 5 6 13 -11 31 2 4 -4 10 10 22 -18 52

     0 0 0 0 1 | 7 33 10 56

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1 0 0 0 0 | 1 6  -2 1 0 0 0 | 0 1 4 -1 1 0 0 | 0 0 -3 -1 0 1 0 | 0 0 2 0 0 0 10 2 0 0

     -25 9 0 0 1 | 0 0 -4 4

= [E|R]

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N(AT) = span1

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4 -1 1 0

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-3 -1 ,

0

l 1 0

6. (15%) (1)

A =

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1 -1 1 0 0 -1 1 0 0 1 0 -1 1 0 0 0 0 0 -1 -1

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3

2 0 0 0 0 0 0 1 0 0 0 0

,C =

0 0 2 0 0 0 0 0 0 1 0 0 0 0 0 0 2 0 0 0 0 0 0 1

(2)

ATCAx =

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1 -1 1 0 0 -1 1 0 0 1 0 -1 1 0 0 0 0 0 -1 -1

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1 -1 0 0 1 0 -1 0 0 -1 1 0 1 0 0 -1 0 1 0 -1 0 0 1 -1

0 0 0 0 0 1

0 0 1 -1

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=

x

1 x

2 x

3 x

4

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Ground node four, then the equation is reduced to:

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x

1

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1 x

2 x

3

=

0 0 -1 -2 -1 4

x

4

0

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→

4  -2 -2 6 -1 -2

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x

1 x

2

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1  =

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0 -1 -2 4

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x

3

0

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4 -2 -1 1 

-2 6 -2 0 -1 -2 4 0

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1 -0.5 -0.25 0.25 

0 1 -0.5 0.1

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0 1 0 0 0 0.4

1 

0.2

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0 1 0 0.2

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→x =

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4

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-0.4 0.4 0.2 0.2

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-0.2 -0 -0.4 0

-0.4 -0.2 ,y = -CAx =